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$$1. \int a \, dx = ax$$

$$2. \int a \cdot f(x) \, dx = a \int f(x) \, dx$$

$$3. \int \phi(y) \, dx = \int \frac{\phi(y)}{y'} \, dy, \quad \text{where } y' = \frac{dy}{dx}$$

$$4. \int (u + v) \, dx = \int u \, dx + \int v \, dx, \quad \text{where } u \text{ and } v \text{ are any functions of } x$$

$$5. \int u \, dv = u \int dv - \int v \, du = uv - \int v \, du$$

$$6. \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$7. \int x^n \, dx = \frac{x^{n+1}}{n+1}, \quad \text{except } n = -1$$

$$8. \int \frac{f'(x) \, dx}{f(x)} = \log f(x), \quad (df(x) = f'(x) \, dx)$$

$$9. \int \frac{dx}{x} = \log x$$

$$10. \int \frac{f'(x) \, dx}{2\sqrt{f(x)}} = \sqrt{f(x)}, \quad (df(x) = f'(x) \, dx)$$

$$11. \int e^x \, dx = e^x$$

$$12. \int e^{ax} \, dx = e^{ax}/a$$

$$13. \int b^{ax} \, dx = \frac{b^{ax}}{a \log b}, \quad (b > 0)$$

$$14. \int \log x \, dx = x \log x - x$$

$$15. \int a^x \log a \, dx = a^x, \quad (a > 0)$$

$$16. \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

INTEGRALS (Continued)

$$17. \int \frac{dx}{a^2 - x^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \frac{x}{a} \\ \text{or} \\ \frac{1}{2a} \log \frac{a+x}{a-x}, \end{cases} \quad (a^2 > x^2)$$

$$18. \int \frac{dx}{x^2 - a^2} = \begin{cases} -\frac{1}{a} \coth^{-1} \frac{x}{a} \\ \text{or} \\ \frac{1}{2a} \log \frac{x-a}{x+a}, \end{cases} \quad (x^2 > a^2)$$

$$19. \int \frac{dx}{\sqrt{a^2 - x^2}} = \begin{cases} \sin^{-1} \frac{x}{|a|} \\ \text{or} \\ -\cos^{-1} \frac{x}{|a|}, \end{cases} \quad (a^2 > x^2)$$

$$20. \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2})$$

$$21. \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{|a|} \sec^{-1} \frac{x}{a}$$

$$22. \int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a} \log \left(\frac{a + \sqrt{a^2 \pm x^2}}{x} \right)$$

FORMS CONTAINING $(a + bx)$

For forms containing $a + bx$, but not listed in the table, the substitution $u = \frac{a + bx}{x}$ may prove helpful.

$$23. \int (a + bx)^n dx = \frac{(a + bx)^{n+1}}{(n+1)b}, \quad (n \neq -1)$$

$$24. \int x(a + bx)^n dx = \frac{1}{b^2(n+2)}(a + bx)^{n+2} - \frac{a}{b^2(n+1)}(a + bx)^{n+1}, \quad (n \neq -1, -2)$$

$$25. \int x^2(a + bx)^n dx = \frac{1}{b^3} \left[\frac{(a + bx)^{n+3}}{n+3} - 2a \frac{(a + bx)^{n+2}}{n+2} + a^2 \frac{(a + bx)^{n+1}}{n+1} \right]$$

$$26. \int x^m(a+bx)^n dx = \begin{cases} \frac{x^{m+1}(a+bx)^n}{m+n+1} + \frac{an}{m+n+1} \int x^m(a+bx)^{n-1} dx \\ \text{or} \\ \frac{1}{a(n+1)} \left[-x^{m+1}(a+bx)^{n+1} \right. \\ \quad \left. + (m+n+2) \int x^m(a+bx)^{n+1} dx \right] \\ \text{or} \\ \frac{1}{b(m+n+1)} \left[x^m(a+bx)^{n+1} - ma \int x^{m-1}(a+bx)^n dx \right] \end{cases}$$

$$27. \int \frac{dx}{a+bx} = \frac{1}{b} \log(a+bx)$$

$$28. \int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$29. \int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}$$

$$30. \int \frac{x dx}{a+bx} = \begin{cases} \frac{1}{b^2} [a+bx - a \log(a+bx)] \\ \text{or} \\ \frac{x}{b} - \frac{a}{b^2} \log(a+bx) \end{cases}$$

$$31. \int \frac{x dx}{(a+bx)^2} = \frac{1}{b^2} \left[\log(a+bx) + \frac{a}{a+bx} \right]$$

$$32. \int \frac{x dx}{(a+bx)^n} = \frac{1}{b^2} \left[\frac{-1}{(n-2)(a+bx)^{n-2}} + \frac{a}{(n-1)(a+bx)^{n-1}} \right], \quad n \neq 1, 2$$

$$33. \int \frac{x^2 dx}{a+bx} = \frac{1}{b^3} \left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2 \log(a+bx) \right]$$

$$34. \int \frac{x^2 dx}{(a+bx)^2} = \frac{1}{b^3} \left[a+bx - 2a \log(a+bx) - \frac{a^2}{a+bx} \right]$$

$$35. \int \frac{x^2 dx}{(a+bx)^3} = \frac{1}{b^3} \left[\log(a+bx) + \frac{2a}{a+bx} - \frac{a^2}{2(a+bx)^2} \right]$$

$$36. \int \frac{x^2 dx}{(a+bx)^n} = \frac{1}{b^3} \left[\frac{-1}{(n-3)(a+bx)^{n-3}} \right. \\ \left. + \frac{2a}{(n-2)(a+bx)^{n-2}} - \frac{a^2}{(n-1)(a+bx)^{n-1}} \right], \quad n \neq 1, 2, 3$$

$$37. \int \frac{dx}{x(a+bx)} = -\frac{1}{a} \log \frac{a+bx}{x}$$

$$38. \int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2} \log \frac{a+bx}{x}$$

$$39. \int \frac{dx}{x(a+bx)^3} = \frac{1}{a^3} \left[\frac{1}{2} \left(\frac{2a+bx}{a+bx} \right)^2 + \log \frac{x}{a+bx} \right]$$

$$40. \int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2} \log \frac{a+bx}{x}$$

$$41. \int \frac{dx}{x^3(a+bx)} = \frac{2bx-a}{2a^2x^2} + \frac{b^2}{a^3} \log \frac{x}{a+bx}$$

$$42. \int \frac{dx}{x^2(a+bx)^2} = -\frac{a+2bx}{a^2x(a+bx)} + \frac{2b}{a^3} \log \frac{a+bx}{x}$$

FORMS CONTAINING $c^2 \pm x^2, x^2 - c^2$

$$43. \int \frac{dx}{c^2 + x^2} = \frac{1}{c} \tan^{-1} \frac{x}{c}$$

$$44. \int \frac{dx}{c^2 - x^2} = \frac{1}{2c} \log \frac{c+x}{c-x}, \quad (c^2 > x^2)$$

$$45. \int \frac{dx}{x^2 - c^2} = \frac{1}{2c} \log \frac{x-c}{x+c}, \quad (x^2 > c^2)$$

$$46. \int \frac{x dx}{c^2 \pm x^2} = \pm \frac{1}{2} \log (c^2 \pm x^2)$$

$$47. \int \frac{x dx}{(c^2 \pm x^2)^{n+1}} = \mp \frac{1}{2n(c^2 \pm x^2)^n}$$

$$48. \int \frac{dx}{(c^2 \pm x^2)^n} = \frac{1}{2c^2(n-1)} \left[\frac{x}{(c^2 \pm x^2)^{n-1}} + (2n-3) \int \frac{dx}{(c^2 \pm x^2)^{n-1}} \right]$$

$$49. \int \frac{dx}{(x^2 - c^2)^n} = \frac{1}{2c^2(n-1)} \left[-\frac{x}{(x^2 - c^2)^{n-1}} - (2n-3) \int \frac{dx}{(x^2 - c^2)^{n-1}} \right]$$

$$50. \int \frac{x dx}{x^2 - c^2} = \frac{1}{2} \log (x^2 - c^2)$$

$$51. \int \frac{x dx}{(x^2 - c^2)^{n+1}} = -\frac{1}{2n(x^2 - c^2)^n}$$

$$517. \int e^x dx = e^x$$

$$518. \int e^{-x} dx = -e^{-x}$$

$$519. \int e^{ax} dx = \frac{e^{ax}}{a}$$

$$520. \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$521. \int x^m e^{ax} dx = \begin{cases} \frac{x^m e^{ax}}{a} - \frac{m}{a} \int x^{m-1} e^{ax} dx \\ \text{or} \\ e^{ax} \sum_{r=0}^m (-1)^r \frac{m! x^{m-r}}{(m-r)! a^{r+1}} \end{cases}$$

$$522. \int \frac{e^{ax}}{x} dx = \log x + \frac{ax}{1!} + \frac{a^2 x^2}{2 \cdot 2!} + \frac{a^3 x^3}{3 \cdot 3!} + \dots$$

$$523. \int \frac{e^{ax}}{x^m} dx = -\frac{1}{m-1} \frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1} \int \frac{e^{ax}}{x^{m-1}} dx$$

$$524. \int e^{ax} \log x dx = \frac{e^{ax} \log x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$$

$$525. \int \frac{dx}{1+e^x} = x - \log(1+e^x) = \log \frac{e^x}{1+e^x}$$

$$526. \int \frac{dx}{a+be^{px}} = \frac{x}{a} - \frac{1}{ap} \log(a+be^{px})$$

$$527. \int \frac{dx}{ae^{mx} + be^{-mx}} = \frac{1}{m\sqrt{ab}} \tan^{-1} \left(e^{mx} \sqrt{\frac{a}{b}} \right), \quad (a > 0, b > 0)$$

$$528. \int \frac{dx}{ae^{mx} - be^{-mx}} = \begin{cases} \frac{1}{2m\sqrt{ab}} \log \frac{\sqrt{a} e^{mx} - \sqrt{b}}{\sqrt{a} e^{mx} + \sqrt{b}} \\ \text{or} \\ \frac{-1}{m\sqrt{ab}} \tanh^{-1} \left(\sqrt{\frac{a}{b}} e^{mx} \right), \quad (a > 0, b > 0) \end{cases}$$

$$529. \int (a^x - a^{-x}) dx = \frac{a^x + a^{-x}}{\log a}$$

$$530. \int \frac{e^{ax}}{b + ce^{ax}} dx = \frac{1}{ac} \log (b + ce^{ax})$$

$$531. \int \frac{x e^{ax}}{(1 + ax)^2} dx = \frac{e^{ax}}{a^2(1 + ax)}$$

$$532. \int x e^{-x^2} dx = -\frac{1}{2} e^{-x^2}$$

$$533. \int e^{ax} [\sin (bx)] dx = \frac{e^{ax}[a \sin (bx) - b \cos (bx)]}{a^2 + b^2}$$

$$534. \int e^{ax} [\sin (bx)] [\sin (cx)] dx = \frac{e^{ax}[(b - c) \sin (b - c)x + a \cos (b - c)x]}{2[a^2 + (b - c)^2]} \\ - \frac{e^{ax}[(b + c) \sin (b + c)x + a \cos (b + c)x]}{2[a^2 + (b + c)^2]}$$

$$535. \int e^{ax} [\sin (bx)] [\cos (cx)] dx = \left\{ \begin{array}{l} \frac{e^{ax}[a \sin (b - c)x - (b - c) \cos (b - c)x]}{2[a^2 + (b - c)^2]} \\ + \frac{e^{ax}[a \sin (b + c)x - (b + c) \cos (b + c)x]}{2[a^2 + (b + c)^2]} \\ \text{or} \\ \frac{e^{ax}}{\rho} [(a \sin bx - b \cos bx) [\cos (cx - \alpha)] \\ - c(\sin bx) \sin (cx - \alpha)] \\ \text{where} \\ \rho = \sqrt{(a^2 + b^2 - c^2)^2 + 4a^2c^2}, \\ \rho \cos \alpha = a^2 + b^2 - c^2, \quad \rho \sin \alpha = 2ac \end{array} \right.$$

$$536. \int e^{ax} [\sin (bx)] [\sin (bx + c)] dx \\ = \frac{e^{ax} \cos c}{2a} - \frac{e^{ax}[a \cos (2bx + c) + 2b \sin (2bx + c)]}{2(a^2 + 4b^2)}$$

$$537. \int e^{ax} [\sin (bx)] [\cos (bx + c)] dx \\ = \frac{-e^{ax} \sin c}{2a} + \frac{e^{ax}[a \sin (2bx + c) - 2b \cos (2bx + c)]}{2(a^2 + 4b^2)}$$

$$538. \int e^{ax} [\cos (bx)] dx = \frac{e^{ax}}{a^2 + b^2} [a \cos (bx) + b \sin (bx)]$$