Rommel, Frank & Adrain — for your interest

to get from: $$RT \ln \frac{f_i}{y_i p} = \int_0^p \left[\left(\frac{\partial V}{\partial n_i} \right)_{T p n_{j \neq i}} - \frac{RT}{p} \right] dp \qquad (1)$$

to: $$= \int_V^\infty \left[\left(\frac{\partial p}{\partial n_i} \right)_{T V n_{j \neq i}} - \frac{RT}{V} \right] dV - RT \ln Z \qquad (2)$$

Integration is at const T and composition.
Assume only

$$p(V, T, n_1, \cdots n_m) \qquad (3)$$

without a specific functional form

Now in a multicomponent mixture of m species with species i varying, and all other species j held constant then at constant temperature

$$p(V, n_i)_{T n_{j \neq i}} \qquad (4)$$

so that

$$\left[\left(\frac{\partial p}{\partial V} \right)_{n_i} \left(\frac{\partial V}{\partial n_i} \right)_p \left(\frac{\partial n_i}{\partial p} \right)_V \right]_{T n_{j \neq i}} = -1$$

or on solving for $(\partial V/\partial n_i)_{T p n_{j\neq i}}$

$$\left[\left(\frac{\partial V}{\partial n_i}\right)_p\right]_{T n_{j\neq i}} = -\left[\left(\frac{\partial V}{\partial p}\right)_{n_i}\left(\frac{\partial p}{\partial n_i}\right)_V\right]_{T n_{j\neq i}} \qquad (6)$$

Substitute (6) in (1) and get

$$RT \ln \frac{f_i}{y_i p} = \int_0^p -\left[\left(\frac{\partial V}{\partial p}\right)_{n_i}\left(\frac{\partial p}{\partial n_i}\right)_V\right]_{T n_{j\neq i}} dp - \int_0^p \frac{RT}{p} dp \qquad (7)$$

$$= -\int_0^p \left(\frac{\partial p}{\partial n_i}\right)_{T V n_{j\neq i}} dV - \int_0^p \frac{RT}{p} dp \qquad (8)$$

let $\frac{RT}{p} = \frac{V}{nZ}$ $\qquad dp = d\left(\frac{nRTZ}{V}\right)$

$$= -\int_0^p \left(\frac{\partial p}{\partial n_i}\right)_{T V n_{j\neq i}} dV - \int_0^p \frac{V}{nZ}\, d\left(\frac{nRTZ}{V}\right)$$

$$= -\int_0^p \left(\frac{\partial p}{\partial n_i}\right)_{T V n_{j\neq i}} dV - \int_0^p \frac{V nRT}{nZ}\left[\frac{V\,dZ - Z\,dV}{V^2}\right]$$

$$= -\int_0^p \left(\frac{\partial p}{\partial n_i}\right)_{TVn_{j\neq i}} dv - \int_0^p \frac{vRT}{z}\left(\frac{1}{v}dz - \frac{Z}{v^2}dv\right)$$

$$= -\int_0^p \left(\frac{\partial p}{\partial n_i}\right)_{TVn_{j\neq i}} dv + \int_0^p \frac{RT}{v}dv - \int_0^p \frac{RT}{Z}dZ$$

gather and change the limits

as $p \to 0$ $v \to \infty$ and $Z \to 1$

$$= -\int_\infty^V \left[\left(\frac{\partial p}{\partial n_i}\right)_{TVn_{j\neq i}} + \frac{RT}{V}\right] dv - \int_1^Z \frac{RT}{Z}dZ$$

$$= \int_V^\infty \left[\left(\frac{\partial p}{\partial n_i}\right)_{TVn_{j\neq i}} - \frac{RT}{V}\right] dv - RT\ln Z$$

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