Exam II

Solution

I. $\ln \gamma_1 = x_2^2 (0.273 + 0.096 x_1)$

$\ln \gamma_2 = x_1^2 (0.273 + 0.096 x_2)$

$\frac{G^E}{RT} = x_1 \ln \gamma_1 + x_2 \ln \gamma_2 = x_1 x_2^2 (0.273 + 0.096 x_1) + x_1^2 x_2 (0.273 - 0.096 x_2)$

$\frac{G^E}{RT} = 0.273 x_1 x_2^2 + 0.273 x_1^2 x_2 + 0.096 x_1 x_2^2 - 0.096 x_1^2 x_2^2$

$G^E/RT = 0.273 (x_1 + x_2)(x_1 x_2)$

a) $\boxed{G^E/RT = 0.273 x_1 x_2}$

b) since this is regular solution w/ $A = 0.273$

$\boxed{\ln \gamma_1 = 0.273 x_2^2 \qquad \ln \gamma_2 = 0.273 x_1^2}$

c) original equations are not thermodynamically consistent.

differential test:

$$x_1 \frac{d \ln \gamma_1}{dx_1} = x_2 \frac{d \ln \gamma_2}{dx_2}$$

$$\frac{\partial \ln \gamma_1}{\partial x_1} = x_2^2 (0.096) + (0.273 + 0.096 x_1)(-2 x_2)$$

$$\frac{\partial \ln \gamma_2}{\partial x_2} = x_1^2 (-0.096) + (0.273 - 0.096 x_2)(-2 x_1)$$

$x_1 [0.096 x_2^2 - 2 x_2 (0.273 + 0.096 x_1)] \stackrel{?}{=} x_2 [-0.096 x_1^2 - 2 x_1 (0.273 - 0.096 x_2)]$

$0.096 x_1 x_2^2 - 2 x_1 x_2 (0.273) - 2 x_1^2 x_2 (0.096) \stackrel{?}{=}$

$-0.096 x_1^2 x_2 - 2 x_1 x_2 (0.273) - 2 x_1 x_2^2 (0.096)$

$0.096 x_1 x_2^2 - 2 x_1 x_2 (0.096) \stackrel{?}{=} -0.096 x_1 - 2 x_2 (0.096)$

$x_2 - 2 x_1 \stackrel{?}{=} -x_1 - 2 x_2$

$(1 - x_1) - 2 x_1 \stackrel{?}{=} -x_1 - 2 (1 - x_1)$

$1 - 3 x_1 \stackrel{?}{=} -2 + x_1$ NO

Integral Test: $\int_0^1 \ln\left(\frac{\gamma_1}{\gamma_2}\right) dx_1 = 0$?

$$\ln\left(\frac{\gamma_1}{\gamma_2}\right) = \ln\gamma_1 - \ln\gamma_2 = x_2^2(0.273 + 0.096x_1) - x_1^2(0.273 - 0.096x_2)$$

$$= 0.273(x_2^2 - x_1^2) + 0.096 x_1 x_2^2 + 0.096 x_1^2 x_2$$

$$= 0.273(1 - 2x_1) + 0.096 x_1 (1 - x_1)$$

$$= 0.273 - 2(0.273)x_1 + 0.096 x_1 - 0.096 x_1^2$$

$$\int \ln\left(\frac{\gamma_1}{\gamma_2}\right) dx_1 = \left[0.273x_1 - 0.273x_1^2 + \frac{0.096}{2}x_1^2 - \frac{0.096}{3}x_1^3\right]_0^1$$

$$\int \ln\left(\frac{\gamma_1}{\gamma_2}\right) dx_1 = 0.273 - 0.273 + \frac{0.096}{2} - \frac{0.096}{3} \neq 0 \quad \underline{\text{No}}$$

II. $\gamma_1 x_1 P_1^s = y_1 P \qquad \gamma_2 x_2 P_2^s = y_2 P$

for az, $x_1 = y_1 \qquad x_2 = y_2$

$\gamma_1^{Az} = \frac{101.3}{77.39} \qquad \gamma_2^{Az} = \frac{101.3}{65.04}$

$\gamma_1^{Az} = 1.309 \qquad \gamma_2^{Az} = 1.5575$

$$A = \ln(1.309)\left[1 + \frac{0.333\ln(1.5575)}{0.667\ln(1.309)}\right]^2 = 0.8934$$

$$B = \ln 1.5575\left(1 + \frac{0.667\ln(1.309)}{0.333\ln(1.5575)}\right)^2 = 2.178$$

when $x_1 = 0.4$, $x_2 = 0.6$

$\gamma_1 = 1.735$ use values w/ A = 0.8934

$\gamma_2 = 1.105$ B = 2.178

$P = \gamma_1 x_1 P_1^s + \gamma_2 x_2 P_2^s = (1.735)(0.4)(52.36) + (1.105)(0.6)(38.98)$

$P = 62.13$ kPa

$y_1 = \frac{\gamma_1 x_1 P_1^s}{62.13} = 0.5848 \qquad y_2 = \frac{\gamma_2 x_2 P_2^s}{62.13} = 0.4152$